Hesham Auda (S'82) was born in Cairo, Egypt, on February 5, 1956. He received the B.Sc. degree from Cairo University, Cairo, Egypt, in 1978, and the M. Eng. degree from McGill University, Montreal, Canada, in 1981. He is currently working toward his Ph.D. degree in the area of numerical solution of electromagnetic field problems.

Roger F. Harrington (S'48–A'53–M'57–SM'62–F'68) was born in Buffalo, NY, on December 24, 1925. He received the B.E.E. and M.E.E. degrees from Syracuse University, Syracuse, NY, in 1948 and 1950, respectively, and the Ph.D. degree from Ohio State University, Columbus, OH, in 1952.

From 1945 to 1946, he served as an Instructor at the U.S. Naval Radio Materiel School, Dearborn, MI, and from 1948 to 1950 he was employed as an Instructor and Research Assistant at Syracuse University. While studying at Ohio State University, he served as a Research Fellow in the Antenna Laboratory. Since 1952 he has been on the faculty of Syracuse University, where he is presently Professor of Electrical Engineering. During 1959–1960 he was Visiting Associate Professor at the University of Illinois, Urbana, in 1964 he was Visiting Professor at the University of California, Berkeley, and in 1969 he was Guest Professor at the Technical University of Denmark, Lyngby, Denmark.

Dr. Harrington is a member of Tau Beta Pi, Sigma Xi, and the American Association of University Professors.

Dielectrically Loaded Corrugated Waveguide: Variational Analysis of a Nonstandard Eigenproblem

ISMO V. LINDELL, SENIOR MEMBER, IEEE, AND ARI H. SIHVOLA, STUDENT MEMBER, IEEE

Abstract —Motivated by simple fabricability, the dielectrically loaded corrugated waveguide is analyzed applying the theory of nonstandard eigenvalues and variational principles recently presented by one of the authors. The eigenvalue parameter of this problem is the boundary susceptance of the corrugated surface, which choice is seen to lead to a simple functional. The functional is tested for the air-filled corrugated guide, and good accuracy for simple test functions is observed. Dispersion relation for the loaded corrugated guide is calculated together with the field pattern for quasi-balanced operation and estimates for the dielectric loss. The method presented here also appears to be applicable in other waveguide problems where inhomogeneous material is involved.

I. Introduction

The CORRUGATED WAVEGUIDE has proven useful for different slow-wave structure applications and for radiating systems requiring rotational symmetry of the power radiation pattern [1]. One of the drawbacks of the corrugated structure is its tedious and costly fabrication. A new method of fabrication was, however, recently suggested by Tiuri,[1] which is quite simple: A dielectric rod is put in a lathe, thin grooves are made on the outside, and the outer surface is metallized. To reduce losses, a hole can be drilled on the axis and we have a dielectrically loaded corrugated waveguide. We are concerned here about the analysis of such a structure.

The conventional air-filled corrugated waveguide can be conveniently analyzed in terms of special functions for the circular cylindrical geometry. The additional dielectric interface, however, makes this approach very complicated. So, a variational method is attempted instead. The eigenvalue problem, however, is not of the standard from $Lf = \lambda Mf$, $Bf = 0$, but of the more general form $L(\lambda)f = 0$, $B(\lambda)f = 0$, i.e., the eigenvalue parameter λ does not appear in the differential equation system in linear form, and it might also be present in the boundary conditions. This more general form of an eigenvalue problem was called a nonstandard eigenvalue problem in recent studies [2], [3], where a variational principle for such problems was also formulated. This method will be applied here. The eigenvalue parameter may be chosen freely among all the parameters of the problem. A stationary functional results if the following functional equation can be solved for the eigenvalue parameter λ:

$$(f, L(\lambda)f) + (f, B(\lambda)f)_b = 0 \tag{1}$$

where the inner products $(\cdot,\cdot), (\cdot,\cdot)_b$ are defined in the domains of the operators L and B, respectively.

Manuscript received August 5, 1982; revised February 17, 1983.

The authors are with the Electrical Engineering Department, Helsinki University of Technology, Otakaari 5A, Espoo 15, Finland 02150.

[1] Helsinki University of Technology, Finland.

0018-9480/83/0700-0520$01.00

A condition for the application of (1) is that the operator pair L, B is self adjoint with respect to the two inner products.

In Section II, the problem is first formulated in terms of the longitudinal components of the electromagnetic field, which leads to a nonstandard eigenvalue problem in all the parameters of the problem. It is seen that if we choose the boundary susceptance of the corrugated surface to be the eigenvalue parameter λ, (1) can be solved easily for λ and a stationary functional for the boundary susceptance is obtained. In Section III, the conventional air-filled corrugated waveguide is considered for reference. Different approximating functions are tested and comparison with exact results found from the literature is made. In Section IV, the dielectrically loaded waveguide is analyzed, and a set of curves describing the dispersion properties of the guide are given. The transverse field pattern is presented for a certain case approximating the self-dual (balanced) operation of the empty guide. Also, losses due to the dielectric loading are estimated with reference to the conductor losses. Finally, the conclusion is given in Section V.

II. The General Inhomogeneous Waveguide

We start by considering the wave propagation problem in a very general waveguide, uniform in the z coordinate direction but possibly inhomogeneous in the transverse plane, and bounded by a surface which may be an anisotropic impedance surface. Thus, the parameters μ and ϵ may be functions of the transverse position vector ρ. Looking for solutions for the fields in the form $\boldsymbol{E}(\rho)e^{-j\beta z}$, $\boldsymbol{H}(\rho)e^{-j\beta z}$, and writing $\boldsymbol{E}$ and $\boldsymbol{H}$ as a sum of the longitudinal components $e(\rho)$, $h(\rho)$ and the transversal components $\boldsymbol{e}(\rho)$, $\boldsymbol{h}(\rho)$, we have from Maxwell's equation

$$\boldsymbol{u}\cdot\nabla\times\boldsymbol{e}+j\omega\mu h=0 \tag{2}$$

$$\nabla e\times\boldsymbol{u}-j\beta\boldsymbol{u}\times\boldsymbol{e}+j\omega\mu\boldsymbol{h}=0 \tag{3}$$

$$\boldsymbol{u}\cdot\nabla\times\boldsymbol{h}-j\omega\epsilon e=0 \tag{4}$$

$$\nabla h\times\boldsymbol{u}-j\beta\boldsymbol{u}\times\boldsymbol{h}-j\omega\epsilon\boldsymbol{e}=0. \tag{5}$$

Here, $\boldsymbol{u}$ is the axial unit vector ($=\boldsymbol{u}_z$) and the transversal fields satisfy $\boldsymbol{u}\cdot\boldsymbol{e}=0$ and $\boldsymbol{u}\cdot\boldsymbol{h}=0$. To reduce the number of unknown quantities, some field components can be eliminated.

For example, we might eliminate the longitudinal components e, h, and the transverse field $\boldsymbol{h}$ to obtain an equation for the transverse electric field alone [4]

$$\nabla\left(\frac{1}{\epsilon}\nabla\cdot(\epsilon\boldsymbol{e})\right)+\mu\boldsymbol{u}\times\nabla\left(\frac{1}{\mu}\boldsymbol{u}\cdot\nabla\times\boldsymbol{e}\right)+(\omega^2\mu\epsilon-\beta^2)\boldsymbol{e}=0. \tag{6}$$

This is an eigenvalue equation of the standard form in both parameters ω^2 and β^2. What is not very convenient is that the operator defined by (6) is not self adjoint, whence a variational formulation would also involve the adjoint problem and the dimension of the problem is doubled [5]. The functional given in [4] is not of a desirable form, because it possesses more stationary points than those corresponding to the solutions of (6).

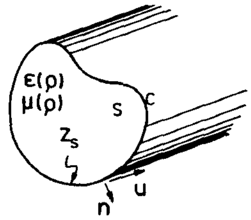


Fig. 1. The inhomogeneous waveguide with anisotropic impedance surface.

Another possibility remains to solve (2)–(5) for the longitudinal components e, h. From (3), (4) we have

$$\begin{pmatrix} e \\ \boldsymbol{u}\times\boldsymbol{h}\end{pmatrix}=-k_c^{-2}\begin{pmatrix} j\beta & -j\omega\mu \\ -j\omega\epsilon & j\beta\end{pmatrix}\begin{pmatrix}\nabla e\\ \boldsymbol{u}\times\nabla h\end{pmatrix} \tag{7}$$

if we define

$$k_c^2(\rho)=\omega^2\mu(\rho)\epsilon(\rho)-\beta^2. \tag{8}$$

Substituting (7) in (2), (4) leaves us with the equation pair

$$\nabla\cdot\left(k_c^{-2}\begin{pmatrix}\omega\epsilon\nabla & -\beta\boldsymbol{u}\times\nabla\\ \beta\boldsymbol{u}\times\nabla & \omega\mu\nabla\end{pmatrix}\begin{pmatrix}e\\h\end{pmatrix}\right)+\begin{pmatrix}\omega\epsilon & 0\\ 0 & \omega\mu\end{pmatrix}\begin{pmatrix}e\\h\end{pmatrix}=0. \tag{9}$$

These equations were derived by Kurtz and Streifer [6] for optical fiber analysis. As can be readily seen, (9) is not of the standard eigenvalue form for either of the parameters ω, β. What makes this formulation attractive for variational analysis is that it defines a self-adjoint problem, as can be shown.

The boundary values were not included in the analysis in [6], because the optical fiber is an open waveguide. Here, we consider a boundary surface defined by a closed curve C on the plane $z=0$ (Fig. 1).

For an anisotropic impedance surface, we can write the boundary conditions in the form

$$\boldsymbol{n}\times\boldsymbol{E}=\boldsymbol{Z}_s\cdot\boldsymbol{H} \tag{10}$$

where $\boldsymbol{Z}_s$ is a two-dimensional dyadic [7], i.e., it satisfies $\boldsymbol{n}\cdot\boldsymbol{Z}_s=\boldsymbol{Z}_s\cdot\boldsymbol{n}=0$, where $\boldsymbol{n}$ is the outer normal unit vector on the curve C, Fig. 1.

For simplicity, we assume a diagonal form for the impedance dyadic

$$\boldsymbol{Z}_s=\boldsymbol{uu}Z_s+(\boldsymbol{n}\times\boldsymbol{u})(\boldsymbol{n}\times\boldsymbol{u})/Y_s \tag{11}$$

which is valid for axial and transverse corrugations but invalid if the corrugations are helical. For an isotropic boundary we have $Y_s=1/Z_s$, but for a corrugated surface Z_s and Y_s are independent. In fact, for ideal transverse corrugations we have $Z_s=0$ and Y_s may take on any value depending on the depth of the corrugations.

Substituting (11) in (10) and separating the axial and transverse field components, gives us the boundary conditions in the form

$$\boldsymbol{n}\cdot\boldsymbol{u}\times\boldsymbol{e}=-Z_s h \tag{12}$$

$$\boldsymbol{n}\cdot\boldsymbol{u}\times\boldsymbol{h}=Y_s e. \tag{13}$$

For the axial fields alone, we have

$$-\boldsymbol{n}\cdot k_c^{-2}\begin{pmatrix}\omega\epsilon\nabla & -\beta\boldsymbol{u}\times\nabla\\ \beta\boldsymbol{u}\times\nabla & \omega\mu\nabla\end{pmatrix}\begin{pmatrix}e\\ h\end{pmatrix}-j\cdot\begin{pmatrix}Y_s & 0\\ 0 & Z_s\end{pmatrix}\begin{pmatrix}e\\ h\end{pmatrix}=0. \quad (14)$$

Now it can be shown that the operators L and B defined by (9) and (14), respectively, define a self-adjoint pair of operators with respect to the conventional integral definitions of the two inner products

$$(f_1,f_2)=\int_S (e_1e_2+h_1h_2)\,dS \quad (15)$$

$$(f_1,f_2)_b=\oint_C (e_1e_2+h_1h_2)\,dC. \quad (16)$$

In fact, it is not difficult to demonstrate that the following Green's formula [8] is valid:

$$(f_1,Lf_2)+(f_1,Bf_2)_b=(Lf_1,f_2)+(Bf_1,f_2)_b. \quad (17)$$

Thus, we are able to apply the general variational principle (1), which gives us an equation between the different parameters of the problem and the field quantities $e(\rho)$, $h(\rho)$. If one of the parameters can be solved from (1), a stationary functional is obtained for that parameter according to the theory [2]. Equation (1) can be written in the form

$$\int_S\left[k_c^{-2}\left(\omega\epsilon(\nabla e)^2+2\beta\boldsymbol{u}\cdot\nabla e\times\nabla h+\omega\mu(\nabla h)^2\right)+\left(\omega\epsilon e^2+\omega\mu h^2\right)\right]dS+j\oint_C\left(Y_se^2+Z_sh^2\right)dC=0. \quad (18)$$

A variational expression analogous to (18) without boundary terms was given recently in [9] for inhomogeneous optical waveguides, but with a different coefficient of the term $\boldsymbol{u}\cdot\nabla e\times\nabla h$. From the evidence of our results we believe that (18) is correct. For certain test functions, the term in question may be zero and thus have no effect on the calculations, as may have happened in [9].

Equation (18) is a very complicated equation in the parameters ω and β if the medium is not homogeneous, because the term k_c^{-2} involves both parameters. We are not, however, limited to those parameters in choosing our eigenvalue parameter λ. In fact, also Y_s and Z_s may be applied. If the boundary impedance were isotropic so that $Y_s=1/Z_s$, (18) is an algebraic equation of the second degree in Z_s or Y_s, and it can be readily solved to obtain a stationary functional for Y_s or Z_s. So, we are able to handle inhomogeneous waveguides with lossy boundaries with this technique.

For a corrugated surface, we may approximate the boundary impedance by $Z_s=0$ and the only remaining parameter is Y_s, which depends on the depth of the corrugations. In this case, (18) reduces to a linear equation in Y_s, which can be solved

$$Y_s=\frac{j\omega}{\oint_C e^2\,dC}\int_S\left[k_c^{-2}\left(\epsilon(\nabla e)^2+2\frac{\beta}{\omega}\boldsymbol{u}\cdot\nabla e\times\nabla h+\mu(\nabla h)^2\right)-\epsilon e^2-\mu h^2\right]dS. \quad (19)$$

That (19) really is a stationary functional for the solutions of (9), (14) with $Z_s=0$ can be readily checked. Equation (19) is the basis of the present method for the loaded corrugated waveguide. Inserting approximations for the fields e,h for fixed values of the parameters ω,β, approximations for the boundary admittance Y_s are obtained. For the known relation between Y_s and s, the depth of the corrugations, there results a relation between the parameters β, ω, and s. If we could solve the parameter β from (19), a stationary functional would result, as demonstrated in [2].

III. The Corrugated Circular Waveguide

In this study, we only consider a circular cylindrical geometry and a step inhomogeneity of the dielectric parameter, as provided by the suggested fabrication procedure. Because the present method is intended for a programmable calculator, we have to apply suitable approximations for the corrugated surface admittance function.

A. Approximations for the Corrugated Surface

In the structure depicted in Fig. 2, we approximate the corrugated surface by an anisotropic impedance surface with radius b.

If the period t of the corrugations is small enough, a radial TEM admittance seen from the boundary C is a good approximation for the surface admittance Y_s. The thickness of the corrugating metal disks is assumed very small compared with t. The condition $t/\lambda<0.2$ given in [10] for plane structure is probably valid also here. The exact TEM admittance can be written for fields possessing $\cos\phi$ or $\sin\phi$ variance on the azimuth angle [11], [12]

$$Y_s=jB_s=\frac{j}{\eta}\frac{J_1'(kb)Y_1(kd)-Y_1'(kb)J_1(kd)}{J_1(kb)Y_1(kd)-Y_1(kb)J_1(kd)} \quad (20)$$

valid for any values of b and $d=b+s$. $\eta=\sqrt{\mu/\epsilon}$.

Expression (20) carries the relation between the parameter Y_s, the geometrical parameters b, s, and the dielectric parameter ϵ_r. Because the present method was designed for calculator applications, we applied several approximate formulas for the exact formula (20) without Bessel functions. If the radius b is large, we may apply the large argument asymptotic formulas for the Bessel functions and obtain the simple expression

$$\eta B_s\simeq-\cot(ks)-1/2kb. \quad (21)$$

This is given in incorrect form in [12], where the sign in front of $1/2kb$ term is reversed. If kb is large enough, the

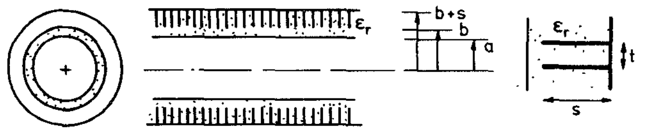


Fig. 2. Dielectrically loaded corrugated circular waveguide.

error is small, and we can write

$$\eta B_s \simeq -\cot(ks). \tag{22}$$

If ks is small, we can approximate Bessel functions by their Taylor expansions, whence with no restrictions on b we have from (20)

$$\eta B_s \simeq -1/ks. \tag{23}$$

Equation (23) also approximates (22), which is only valid for large kb. The approximation (23), however, is also good for small kb, if ks is small enough. All three approximations for (20) will be applied.

B. Approximations for the Fields

To apply a variational method, we must find reasonable approximations for the unknown field functions. We are here interested only in those modes possessing $\cos\phi$ or $\sin\phi$ dependence on ϕ, corresponding to modes designated by EH_{1n} and HE_{1n} in air-filled guides. For the general inhomogeneous circular guide with circular symmetry of the medium, we can see from (9) that an assumption of the separable form

$$e(\rho,\phi) = f(\rho)\cos\phi \tag{24}$$

for the electric field leads to the form

$$h(\rho,\phi) = g(\rho)\sin\phi \tag{25}$$

for the magnetic field. Because the dielectric interface presents no additional condition for the longitudinal fields as long as they are continuous on the boundary, we may choose any continuous functions f, g to approximate the true fields.

From analyticity of the field functions on the axis (i.e., continuity of the fields and their derivatives) we may conclude that the functions f, g are odd in the argument ρ. Hence, a polynomial approximation of the function f must be of the form

$$f(\rho) = \rho + \alpha\rho^3 + \cdots. \tag{26}$$

In the air-filled guide, the functions f and g are necessarily multiples of the Bessel function, and hence, of each other, whence a reasonable approximation of the g function at first is to choose

$$g(\rho) = Af(\rho) \tag{27}$$

where A is a parameter to be determined through the functional. If (26), (27) are inserted in (24), (25), and in the functional (19), we may determine the parameters $A, \alpha, \cdots$ by setting the derivatives of Y_s with respect to these parameters equal to zero, whence equations are obtained. The equations may be too complicated to solve analytically, in which case we may very easily find the stationary points by using a programmable calculator, provided the number of parameters $A, \alpha, \cdots$ is not very high. In fact, with just one or two parameters, an engineering accuracy is easily obtained.

The optimum value of the parameter A is obtainable in analytic form for simplest approximations for the fields. In fact, it is seen from (19) that the stationary point is reached for

$$A_0 = \frac{\beta}{\omega\mu}\frac{\int_0^b [(f^2)'/k_c^2]\,d\rho}{\int_0^b [f^2\rho - (f'^2\rho + f^2/\rho)/k_c^2]\,d\rho} \tag{28}$$

where $k_c^2 = k^2 - \beta^2$ depends on the parameters a and ϵ_r.

For a linear approximation of the function f

$$f_1(\rho) = \rho \tag{29}$$

we may evaluate (28) to obtain the expression

$$A_0 = \frac{\beta}{\omega\mu}\frac{(k_{c1}b)^2 - (k_{c1}^2 - k_{c0}^2)(b^2 - a^2)}{(k_{c1}b)^2\left[\left(\frac{1}{2}k_{c0}b\right)^2 - 1\right] + (k_{c1}^2 - k_{c0}^2)(b^2 - a^2)}. \tag{30}$$

Here we denote $k_{c0}^2 = k_0^2 - \beta^2$ and $k_{c1}^2 = k_1^2 - \beta^2 = \epsilon_r k_0^2 - \beta^2$.

Expression (30) reduces to $\beta/\omega\mu[(k_{c0}b/2)^2 - 1]$ for either $a = b$ or $\epsilon_r = 1$, which both correspond to the homogeneous guide with surface impedance.

More complicated expressions are obtained for more complicated approximations of the f function.

Inserting (30) in (19), an explicit formula for the boundary susceptance is obtained for the linear approximation (29), which however is very complicated in the general form. A special case for the empty corrugated guide will be treated shortly.

C. The Air-Filled Corrugated Guide

To test the accuracy of the present method we consider first the conventional corrugated guide, which has been thoroughly analyzed in [12].

For a linear test function (29), an analytic expression for the dispersion relation is obtained. In fact, setting $\epsilon_r = 1$ in (30) and substituting in (19), we have for the linear approximation

$$\beta b \simeq \sqrt{\frac{\left[(kb)^2 - 4\right]\left[(kb)^2 + 4kb\eta B_s - 4\right]}{kb[kb + 4\eta B_s]}}. \tag{31}$$

Inserting different approximations for the corrugation susceptance B_s: (21), (22), or (23), we have different approximations for the dispersion relation $\beta(\omega, b, s)$. In Figs. 3 and 4, the approximations based on (31) are depicted in dashed line. When comparing to the exact curve taken from [12], we see that the approximation (21) is generally better than (23) except for small values of kb, i.e., for EH_{11} wave. For a test function of the third degree (26), the

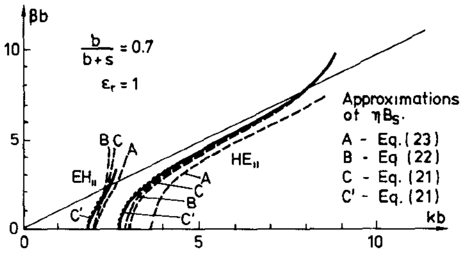


Fig. 3. Different approximations for the dispersion relation of the air-filled corrugated waveguide. Solid line—exact [12]; dashed line—linear approximation; dotted line—cubic approximation. $b/(b+s)=0.7$.

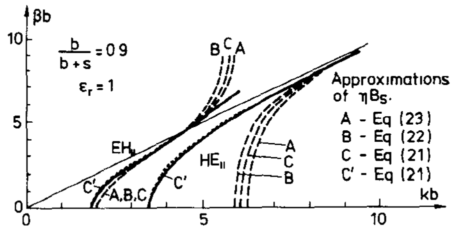


Fig. 4. Different approximations for the dispersion relation of the air-filled corrugated waveguide. Solid line—exact [12]; dashed line—linear approximation; dotted line—cubic approximation. $b/(b+s)=0.9$.

corresponding results are drawn in dotted line, which of course follow the exact curves more closely.

It is noted that the relation $\beta(k)$ is not single-valued for the two modes $EH_{11}-HE_{11}$, whereas the linear approximation (31) is. Therefore, the linear approximation does not work well for one of the modes in the nonuniqueness region. This happens near the cutoff of the HE_{11} mode, as is clearly seen in Fig. 4. The cubic approximation does not lead to a unique expression like (31), whence in the multiple-valued region there exists two stationary points. It is noted that, for this problem, the cubic approximation together with (21) gives us results within the reading accuracy of the exact curve of [12]. Equation (31) is not without value, however. Near balanced operation ($B_s=0$) and TEM operation ($\beta=k$), the linear test function gives us a pretty close approximation with either B or C approximations of the susceptance function, for the dispersion relation of the HE_{11} mode.

IV. The Dielectrically Loaded Corrugated Guide

A. Dispersion Properties

In Fig. 2, the geometry of the dielectrically loaded corrugated guide is given. The dielectric is teflon with $\epsilon_r=2.08$, and dispersion relation was calculated for three values of the thickness of the loading layer, namely for a/b values 0.95, 0.9, and 0.8. Because of the relatively small thickness, the test functions were taken as before, (24), (25), and (27). In the air region, (27) is exactly valid, but in the dielectric it is not and thus introduces some error. We applied cubic test function for $f(\rho)$ in (26) and (21) for the boundary susceptance (corresponding to curves C in Figs. 3 and 4). The results are given in Fig. 5 (a)–(c).

In Fig. 5, only the HE_{11} mode is analyzed. It is seen that

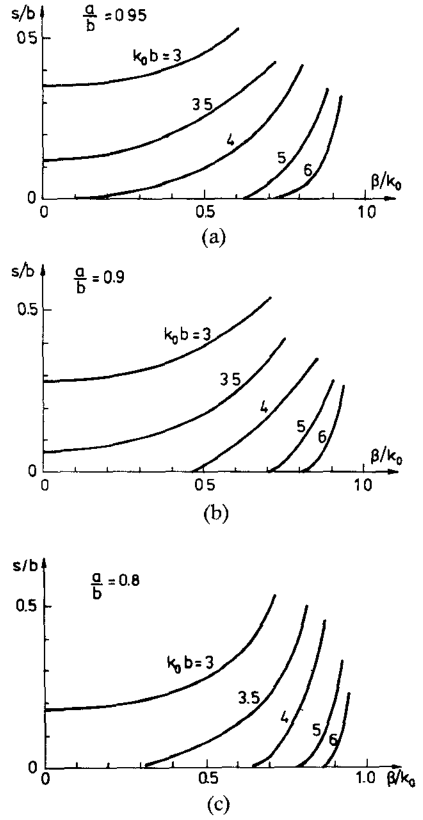


Fig. 5. Dispersion relation for a dielectrically loaded corrugated waveguide for different values of the thickness of the dielectric layer. (a) $a/b=0.95$, (b) $a/b=0.9$, (c) $a/b=0.8$. Dielectric material is teflon, $\epsilon_r=2.08$. Cubic test functions and approximation (21) were applied.

for a certain propagation factor at a certain frequency, the depth of the corrugations needed is smaller if the dielectric layer is made thicker. Because losses increase with increasing dielectric material, this method cannot be extended very far.

B. Fields in Quasi-Balanced Operation

In balanced operation, the air-filled corrugated guide satisfies the condition $B_s=0$, and the ratio of the axial electric and magnetic fields is either η or $-\eta$, depending on the mode. This operation can also be called self dual, because the duality transformation performed on the balanced field leaves the field invariant [13]. This is only possible for problems possessing self-dual media and boundaries, whence a strict self-dual solution does not exist for dielectrically loaded corrugated guides.

For thin dielectric layers, however, there exist modes that do not differ much from a balanced mode in the corresponding empty guide. The quasi-balanced condition of a dielectrically loaded corrugated guide could be defined either by $B_s=0$ or by $A=1/\eta$ to start with. The former definition is not very convenient, because we loose our eigenvalue parameter, and the variational principle should be reformulated. The latter definition, however, is applicable.

As an example, we consider the HE_{11} mode in a corrugated guide partially filled with teflon, with the ratios $a/b=0.9$ and $b/(b+s)=0.7$. The following expression

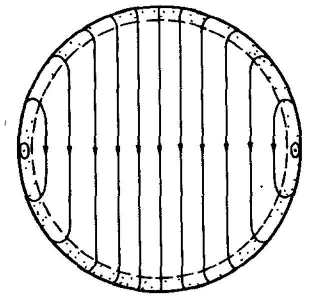


Fig. 6. Sketch of the transverse field pattern for the quasi-balanced HE_{11} mode defined by (33). $a/b = 0.9$, $b/(b+s) = 0.7$, $\epsilon_r = 2.08$. Cubic test functions and (27) were applied.

for the transverse electric field can be written from (7) applying the approximation (27) and (24), (25):

$$e(\rho,\phi) \simeq -jk_c^{-2}\left\{u_x\left(\frac{\beta+k_0\eta A}{2}\right)\left(f'+\frac{f}{\rho}\right) + p\left(\frac{\beta-k_0\eta A}{2}\right)\left(f'-\frac{f}{\rho}\right)\right\}. \quad (32)$$

Here, the unit vector p equals $u_x\cos(2\phi)+u_y\sin(2\phi)$. No assumption on the approximating function $f(\rho)$ has yet been made.

For the linear approximation, we see from (32) that the second term is zero, whence the approximating transverse field has constant polarization for every value of the parameter A.

For the cubic approximation, the condition $A = 1/\eta$ can be found by calculating values of optimum A from (28) along the dispersion curve. In this case, it was found that for the parameter values $k_0 b \simeq 4.0$, $\beta/k_0 \simeq 0.8$, and $\alpha \simeq 0.5/b^2$ we have $A_0 \simeq 1/\eta$. In this operation point, expression (32) gives us the numerical field

$$e = -1.09jb\left\{u_x\left(1-(\rho/b)^2\right)+0.08p(\rho/b)^2\right\}, \quad \text{for } \rho < a$$

$$= -0.33jb\left\{u_x\left(1-(\rho/b)^2\right)+0.08p(\rho/b)^2\right\}, \quad \text{for } \rho > a. \quad (33)$$

A sketch of (33) is given in Fig. 6. It is seen that the field polarization is essentially constant, parallel to u_x, for $\rho < \rho_0$, where ρ_0 is the solution of $(\beta + k_0\eta A)(1-2\alpha\rho^2)+2(\beta - k_0\alpha A)\alpha\rho^2 = 0$. For the present example we have $\rho_0 \simeq 0.96b$. The field intensity is small at the boundary; for (33) we have $e(b)$ values of only 2.2 percent of $e(0)$ value. Thus, the cross polarization should not be very great for radiation.

From (32), we see that for the approximation (27) we can find an operation point with no cross polarization in the transverse field. In fact, taking $A = \beta/k_0\eta$ we see that in (32) the second term disappears for all approximating functions f. This means that the ratio of the longitudinal fields should equate the TE wave impedance. The corresponding operation point can again be found stepping along the dispersion curve and calculating values of A_0. For the previous example, we have $k_0 b \simeq 2.5$, $\beta/k_0 \simeq 0.34$ corresponding to the coefficients $A \simeq 0.34/\eta$, $\alpha \simeq 0.59/b^2$.

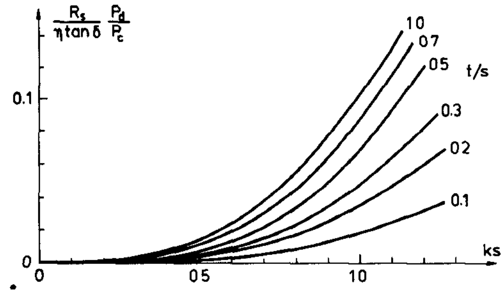


Fig. 7. Normalized ratio of the dielectric and conductor losses in dielectrically loaded corrugated waveguide for $a = b$.

C. Dielectric Losses of the Loaded Guide

Finally, we make a crude estimation of the losses involved in the dielectric loading of the corrugated guide to get an idea whether the suggested structure is applicable. To keep calculations simple, we consider the limit of a thin dielectric layer, i.e., let $b \to a$. Thus, only dielectric losses within the corrugations are calculated. Secondly, we only consider large values of $kb = \sqrt{\epsilon_r}k_0 b$, whence the asymptotic expressions of the Bessel functions can be applied.

For the respective dielectric and conductor power losses of the radial TEM field within one period of the corrugation we can write

$$P_d = \tan\delta\frac{k}{2\eta}\int|E|^2\,dV \quad (34)$$

$$P_c = (R_s/2)\oint|H|^2\,dS \quad (35)$$

where the volume integral is over a period volume inside the corrugations, and the surface integral is taken over the surface of a period of the corrugating material.

We consider the ratio of (34) and (35). For the approximations expressed above we can write

$$P_d/P_c \cong (\eta\tan\delta/R_s)(kt/2)\frac{2ks-\sin 2ks}{2k(t+s)+\sin 2ks}. \quad (36)$$

This is depicted in Fig. 7 in normalized form for different values of ks and the ratio t/s. It is seen that for small s, the dielectric losses are negligible. This is evident, because for small s, the E field in the slot is small, whereas the H field is not.

As a numerical example, we consider a corrugated copper waveguide loaded by teflon dielectric. At 10 GHz we have $\tan\delta \simeq 0.00037$ and $R_s \simeq 26$ mΩ, whence the values in Fig. 7 must be multiplied by 5.4 to obtain the ratio P_d/P_c. It is seen that for all values of ks and t/s in Fig. 7, we have dielectric losses smaller than conductor losses. For $t/s < 0.5$ and $s < \lambda/10$ we have $P_d < 0.1P_c$. Since for HE_{11} operation the air-filled corrugated waveguide has very low losses [14], the dielectric loading can be made without much affecting this property. This conclusion, however, presumes that the dielectric layer is very thin.

V. Conclusion

The theory of nonstandard eigenvalues and their approximate determination through variational methods was applied to the problem of dielectrically loaded corrugated waveguides. The motivation for this analysis was a fabrication process, which appears simpler than that for the air-filled corrugated guide. The functional was derived through longitudinal field components and for the eigenvalue parameter, the boundary susceptance of the corrugated surface was adopted. The functional was seen to give good results for the empty corrugated guide, whence confidence on the new results for the loaded guide could be justified. Finally, quasi-balanced operation and loss estimation were considered. The dielectric losses did not appear too heavy for applicable structures.

Acknowledgment

The authors thank the anonymous referee for leading us to the references [15], [16].

References

[1] H. C. Minnett and B. MacA. Thomas, "A method of synthesizing radiation patterns with axial symmetry," *IEEE Trans. Antennas Propagat.*, vol. AP-14, pp. 654–656, Sept. 1966.

[2] I. V. Lindell, "Variational methods for nonstandard eigenvalue problems in waveguide and resonator analysis," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 1194–1204, Aug. 1982.

[3] I. V. Lindell, "Variational methods for nonstandard eigenvalue problems in microwave field analysis," in *IEEE MTT-S 1982 Microwave Symp. Dig.*, (Dallas, TX), 1982.

[4] K. Kurokawa, "Electromagnetic waves in waveguides with wall impedance," *IRE Trans. Microwave Theory Tech.*, vol. MTT-10, pp. 314–320, Sept. 1962.

[5] G. Jeng and A. Wexler, "Self-adjoint variational formulation of problems having non-self-adjoint operators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 91–94, Feb. 1978.

[6] C. N. Kurtz and W. Streifer, "Guided waves in inhomogeneous focusing media—Part I: Formulation, solution for quadratic inhomogeneity," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 11–15, Jan. 1969.

[7] I. V. Lindell, "Elements of dyadic algebra and its application in electromagnetics," Helsinki Univ. Technology, Radio Lab. Rep. S126, 1981.

[8] J. van Bladel, *Electromagnetic Fields*. New York: McGraw-Hill, 1964.

[9] C. Yeh *et al.*, "Single-mode optical waveguides," *Appl. Optics*, vol. 18, no. 10, pp. 1490–1504, May 15, 1979.

[10] G. L. James, "Surface reactance of corrugated planes," *Electron. Lett.*, vol. 15, no. 23, pp. 751–753, Nov. 8, 1979.

[11] P. J. B. Clarricoats and P. K. Saha, "Theoretical analysis of cylindrical hybrid modes in a corrugated horn," *Electron. Lett.*, vol. 5, pp. 187–189, May 1, 1969.

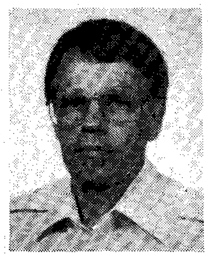
[12] P. J. B. Clarricoats and P. K. Saha, "Propagation and radiation behavior of corrugated feeds—Part I: The corrugated waveguide feed," *Proc. Inst. Elec. Eng.*, vol. 118, no. 9, pp. 1167–1176, Sept. 1971.

[13] I. V. Lindell, "Asymptotic high-frequency modes of homogeneous waveguide structures with impedance boundaries," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 1087–1093, Oct. 1981; Correction, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, p. 296, Mar. 1982.

[14] P. J. B. Clarricoats, A. D. Olver, and S. L. Chong, "Attenuation in corrugated circular waveguides," *Proc. Inst. Elec. Eng.*, vol. 122, no. 11, pp. 1173–1179, Nov. 1975.

[15] M. J. Al-Hakkak and Y. T. Lo, "Circular waveguides with anisotropic walls," *Electron. Lett.*, vol. 6, no. 24, pp. 786–789, Nov. 1970.

[16] M. J. Al-Hakkak, "Dielectric loading of corrugated waveguides," *Electron. Lett.*, vol. 8, no. 7, pp. 179–180, Apr. 1972.



Ismo V. Lindell (S'68–M'70–SM'83) was born in Viipuri (Wyborg), Finland, on November 23, 1939. He received the degrees of Dipl. Eng., Lic. Tech., and Dr. Tech. in 1963, 1967, and 1971, respectively, all in electrical engineering from the Helsinki University of Technology, Espoo, Finland.

From 1963 to 1970 he was a Research and Teaching Assistant, Acting Associate Professor from 1970 and 1975, and Associate Professor since 1975, in Radio Engineering with the Helsinki University of Technology. During the academic year 1972–1973 he was a Visiting Associate Professor at the University of Illinois, Urbana.

Dr. Lindell is the Commission B official member of Finland in URSI. His main interest is in electromagnetic theory.

Ari H. Sihvola (S'80) was born in Valkeala, Finland, on October 6, 1957. He received the degree of Diploma Engineer in electrical engineering from the Helsinki University of Technology, Helsinki, Finland, in 1981, where he is working as a Teaching and Research Assistant. His fields of interest are electromagnetic field theory and remote sensing. He is the present chairman of the IEEE Student Branch at the Helsinki University of Technology.